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PORTFOLIO SELECTION FROM THE CNX NIFTY STOCKS BASED ON SHARPE'S SINGLE INDEX MODEL

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ABSTRACT
The Capital Market is one of the most important and dynamic markets of the Indian Economy. It is a volatile market, where prices constantly move up and down due to the ever-changing forces of demand and supply. Volatility is nothing but the risk that an investor bears, be it an individual investor or an institutional investor. Instead of investing his entire fund in one security, the very concept of portfolio enables the investor to reduce or diversify his risk, without compromising on the returns. Such a portfolio of securities which bears the minimum risk for a given level of return is known as the Minimum Variance Portfolio or the Optimal Portfolio. This optimization is only possible once we have a model of the portfolio return and risk. In this research paper, an attempt has been made to get an overview of the Sharpe's Single Index Model and to construct an optimal portfolio from among the fifty stocks of the CNX Nifty Index. For this, the daily closing price from 3rd May, 2010 to 30th April, 2015 is taken to calculate the returns, beta and various other parameters needed for the study.

KEYWORDS: Sharpe's Single Index Model, Optimal Portfolio, Minimum Variance Portfolio, risk-return trade off, beta, systematic risk, unsystematic risk, cut-off point, excess return to beta ratio.

INTRODUCTION

According to the research work done by William Sharpe, the Modern Portfolio Theory indicates the maximization of returns through a combination of different securities[i]. This theory tells us that risk can be reduced by combining low risk securities with those of high risk. This theory depends upon the concept of diversification. Similar to the traditional theory, the modern portfolio theory assumes that there is free and perfect flow of information and that the market is perfect and absorbs quick information. It is to be noted that the riskiness of a financial asset in a portfolio is to be seen in the context of the market related risk or portfolio risk, but not in isolation.

Portfolio is a combination of securities such as stocks, bonds, money market instruments, etc. The process of blending together the broad asset classes so as to obtain optimum return with minimum risk is called portfolio construction. This diversification helps to spread the risk over many assets. Diversification of securities gives the assurance of obtaining the anticipated return on portfolio. In a diversified portfolio, some securities may not perform as expected, but others may exceed the expectation and make the actual return of the portfolio reasonably close to the anticipated one. Investing in a single security may lead to a greater likelihood of the actual return being different from that of the expected return. Hence it is a common practice to diversify investment among the various securities in the portfolio. The Sharpe's Single Index Model states that if a stock is ranked by excess return to beta (from highest to lowest), the ranking represents the desirability of any stock's inclusion in a portfolio[ii]. Keeping this fact in mind an optimal equity portfolio with diversification of investments in all sectors has been constructed from among the CNX Nifty stocks using the current stock prices.

LITERATURE REVIEW

In the paper titled "Sharpe's Single Index Model And Its Application To Construct Optimal Portfolio: An Empirical Study" by Niranjan Mondal, an attempt has been made to get an insight into the idea embedded in Sharpe's single index model and to construct an optimal portfolio empirically using this model by taking BSE SENSEX as market performance index and considering daily indices along with the daily prices of sampled securities for the period of April 2001 to March 2011. In the paper titled "Optimal Portfolio Construction Using Sharpe's Single Index Model - A Study Of Selected Stocks From BSE" by Dr. R. Nalini, awareness in the minds of investors regarding the utility of Sharpe's Single Index Model in portfolio construction is created and from among the fifteen sample companies of the S&P BSE Sensex index, only four were selected for optimal portfolio using the Sharpe's Single Index Model. In the paper titled "A Study on Usage of Sharpe's Single Index Model in Portfolio Construction With Reference To CNX Nifty" by Ms Apurva A Chauhan, a portfolio is constructed from stocks of CNX nifty by collecting data from the top ten companies of CNX nifty

based on their weights for the time period of August 2014. In the research work done by Suresh Chandra Das and Dr. Bishnupriya Mishra on the topic "Optimal Portfolio- Does Number of scrips matter?", Sharpe Optimal Portfolio technique is used for portfolio construction. In this paper, it is suggested that the analysis of security, portfolio selection and proper management of portfolio will help in enhancement of investor's awareness regarding trend and changes that are present in the market. The paper also suggests that optimum portfolio helps in minimizing the risk, without compromising on return. It is to be noted that all the above mentioned research papers are based on the stock prices of their respective periods. The equity market is always a dynamic ever changing environment for equity investors. Hence, the construction of an optimal portfolio based on the latest prices of the NIFTY stocks and that of the index itself is essential for accurate results and to suggest an optimal equity portfolio for any individual or institutional investor.

OBJECTIVES OF THE STUDY

The objectives of this study are as follows:

- To gauge the importance of portfolio management.
- To analyse how to reduce the risk of the portfolio through diversification, keeping in mind its trade-off with the return.
- To recommend an optimal equity portfolio (using the latest stock prices and other parameters) to any individual or institutional investor, for the betterment of investment results.

Assumptions of the Study

- All equity investors have homogeneous expectations.
- The risk-return estimations of a security are carried out in a uniform holding period (3rd May, 2010 to 30th April, 2015).
- The relationships between the securities are primarily due to their individual influences along with business and economic factors.
- The covariance between the security returns can be explained by a single factor.
- The covariances among the securities are due to varying responses to the macroeconomic factors.

Sharpe's Single Index Model

The construction of an optimal portfolio is simplified if a single number measures the desirability of including a stock in the optimal portfolio. If we accept the Sharpe's Single Index Model, such a number exists. In this case, the desirability of any stock is directly related to (depend upon) its excess return to beta ratio:

$$\frac{Ri - Rf}{\beta i}$$

Where:
R_i = return on the ith stock
R_f = return on risk free asset
β_i = beta of the ith stock

The model explicitly implies that if securities are ranked in descending order on the basis of their excess return-to-beta ratio, the ranking represents the desirability of including a security in the portfolio.

METHODOLOGY AND RESEARCH DESIGN

Secondary data source is used in this research work, where quantitative research techniques are used by choosing data for the 50 companies which are part of the CNX Nifty Index. The sample size being 50 (>30) indicates a large sample has been used in the research and hence is likely to give accurate results. The stock price movement (closing price daily) from 3rd May, 2010 to 30th April, 2015 is taken and the risk free rate of return (R_f) is taken as 8.13% which is calculated by taking the monthly average of 10 year Government Bond for last one year. Given these, certain financial parameters like β for each stock, the stock return, systematic & unsystematic risk and the Sharpe's Single Index are calculated. In order to determine the stocks which are to be included in the optimum equity portfolio, the cut off point was found out to determine the inclusion of a stock in the optimal equity portfolio. Finally, the proportion of each stock in the equity portfolio was calculated in order to get the values of the portfolio return, portfolio β and the portfolio variance (portfolio risk).

Calculation of returns

The daily closing price of the Nifty Fifty stocks and the Index itself were recorded from 3rd May, 2010 to 30th April, 2015 and the natural logarithmic returns (continuously compounded return) were calculated for each day for each of the 50 stocks and for the market, using the formula:

$$\text{Return}_i = \ln\left[\frac{Pi}{P(i-1)}\right]$$

Where, P_i is the closing price of the ith day and P_{i-1} is the closing price of the previous day.

Logarithmic returns were taken to smoothen out the daily price fluctuations and to make conversion to average annual returns easily by multiplying the average daily returns with 365.

Calculation of Beta (β)

Beta for each stock was calculated by using the formula

$$\beta = \frac{\text{Covariance of Stock return w.r.t Market return}}{\text{Variance of Market return}}$$

Calculation of Total, Systematic & Unsystematic Risk

Variance of stock return (daily), which was used to calculate β was converted to Variance of stock return (annually). This represents the Total Risk.

$$\text{Total Risk} = \text{Var}[\text{Stock Return(annually)}]$$

Systematic Risk

Systematic risk is also known as "Undiversifiable Risk," "Volatility" or "Market Risk". It affects the overall market, not just a particular stock or industry. This type of risk is both unpredictable and unavoidable. It cannot be mitigated through diversification.

For calculation of Systematic Risk, Variance of market return (daily), which was used to calculate β was converted to Variance of market return (annually).

Systematic Risk is given by the formula:

$$\text{Systematic Risk} = \beta^2\sigma_m^2$$

Unsystematic Risk: Unsystematic risk is also known as "Non-systematic Risk," "Specific Risk," "Diversifiable Risk" or "Residual Risk". This type of risk can be reduced through diversification.

$$\text{Unsystematic Risk } (\sigma_{ei}^2) = \text{Total Risk} - \text{Systematic Risk}$$

DATA ANALYSIS AND INTERPRETATION

The risk analysis can be explained in the following table (Table -1).

Company Name	Covariance between market return and stock return (daily) (%)	Variance of market return (daily) (%)	Beta (β)	Variance of Stock return (daily) (%)	Variance of Stock return (annually) (Total Risk) (%)	Variance of market return (annually) (%)	Systematic risk	Unsystematic risk
ACC	0.009	0.011	0.814	0.027	9.736	4.133	2.738	6.997
Ambuja Cem.	0.011	0.011	0.951	0.040	14.493	4.133	3.739	10.754
Asian Paints	0.006	0.011	0.567	0.026	9.534	4.133	1.328	8.206
Axis Bank	0.017	0.011	1.494	0.049	17.956	4.133	9.230	8.726
B H E L	0.014	0.011	1.242	0.059	21.459	4.133	6.374	15.085
B P C L	0.009	0.011	0.821	0.045	16.416	4.133	2.785	13.631
Bajaj Auto	0.008	0.011	0.690	0.025	9.100	4.133	1.966	7.134
Bank of Baroda	0.013	0.011	1.173	0.047	17.279	4.133	5.684	11.595
Bharti Airtel	0.009	0.011	0.838	0.041	14.868	4.133	2.905	11.963
Cairn India	0.009	0.011	0.758	0.032	11.768	4.133	2.373	9.394
Cipla	0.006	0.011	0.518	0.023	8.505	4.133	1.109	7.396
Coal India	0.008	0.011	0.669	0.035	12.935	4.150	1.860	11.075
Dr Reddy's Labs	0.005	0.011	0.454	0.021	7.656	4.133	0.851	6.805
GAIL (India)	0.008	0.011	0.673	0.028	10.199	4.133	1.871	8.328
Grasim Inds	0.008	0.011	0.744	0.028	10.206	4.133	2.288	7.918
Housing Dev.	0.013	0.011	1.118	0.030	11.125	4.133	5.170	5.955
HCL Tech.	0.008	0.011	0.743	0.035	12.892	4.133	2.283	10.608
HDFC Bank	0.012	0.011	1.041	0.023	8.448	4.133	4.478	3.970
Hero Motocorp	0.007	0.011	0.624	0.033	12.060	4.133	1.607	10.453
Hind. Unilever	0.005	0.011	0.464	0.025	9.079	4.133	0.890	8.190
Hindalco Inds.	0.017	0.011	1.488	0.064	23.281	4.133	9.145	14.136
I D F C	0.018	0.011	1.558	0.059	21.575	4.133	10.034	11.542
ICICI Bank	0.017	0.011	1.530	0.042	15.262	4.133	9.674	5.587
Idea Cellular	0.007	0.011	0.647	0.050	18.172	4.133	1.730	16.442
IndusInd Bank	0.014	0.011	1.275	0.048	17.585	4.133	6.719	10.866
Infosys	0.008	0.011	0.723	0.033	12.017	4.133	2.158	9.858
ITC	0.008	0.011	0.663	0.023	8.354	4.133	1.818	6.536
Kotak Mah. Bank	0.013	0.011	1.139	0.035	12.655	4.133	5.360	7.295
Larsen & Toubro	0.015	0.011	1.325	0.039	14.167	4.133	7.251	6.916
Lupin	0.005	0.011	0.421	0.026	9.379	4.133	0.733	8.646

M & M	0.011	0.011	0.951	0.034	12.392	4.133	3.734	8.658
Maruti Suzuki	0.009	0.011	0.801	0.033	11.889	4.133	2.652	9.237
NMDC	0.010	0.011	0.896	0.039	14.198	4.133	3.315	10.883
NTPC	0.009	0.011	0.764	0.028	10.155	4.133	2.410	7.745
O N G C	0.010	0.011	0.866	0.035	12.867	4.133	3.102	9.765
Power Grid	0.007	0.011	0.625	0.020	7.307	4.133	1.612	5.694
Punjab Natl.Bank	0.014	0.011	1.213	0.046	16.702	4.133	6.079	10.623
Reliance Inds.	0.013	0.011	1.112	0.028	10.222	4.133	5.106	5.116
St Bk of India	0.014	0.011	1.280	0.040	14.635	4.133	6.775	7.860
Sun Pharma	0.006	0.011	0.542	0.028	10.097	4.133	1.213	8.884
Tata Motors	0.016	0.011	1.382	0.054	19.825	4.133	7.897	11.928
Tata Power Co.	0.011	0.011	0.980	0.040	14.681	4.133	3.966	10.715
Tata Steel	0.015	0.011	1.356	0.048	17.405	4.133	7.596	9.809
TCS	0.008	0.011	0.716	0.029	10.480	4.133	2.116	8.363
Tech Mahindra	0.008	0.011	0.667	0.037	13.532	4.133	1.841	11.690
UltraTech Cem.	0.008	0.011	0.722	0.028	10.353	4.133	2.157	8.196
Vedanta	0.015	0.011	1.319	0.067	24.292	4.133	7.193	17.099
Wipro	0.007	0.011	0.627	0.029	10.705	4.133	1.623	9.082
Yes Bank	0.018	0.011	1.621	0.069	25.253	4.133	10.856	14.397
Zee Entertainment	0.008	0.011	0.666	0.040	14.452	4.133	1.836	12.616

This table specifically portrays how the total risk for each stock is split into systematic and unsystematic risk.

Calculation of Sharpe's Single Index

The average of the daily stock returns were calculated for each stock which was then converted to average stock return annually (R_i) by multiplying by 365.

Risk free rate of return (R_f) is taken as 8.13% which is calculated by taking the monthly average of 10 year Government Bond for last one year as shown in the table (Table 2):

Date	Price
May 01, 2015	7.816
Apr 01, 2015	7.860
Mar 01, 2015	7.736
Feb 01, 2015	7.728
Jan 01, 2015	7.693
Dec 01, 2014	7.857
Nov 01, 2014	8.087
Oct 01, 2014	8.277
Sep 01, 2014	8.516
Aug 01, 2014	8.565
Jul 01, 2014	8.722
Jun 01, 2014	8.747
Average	8.134

Next, the Sharpe's Single Index was calculated using the formula:

$$\frac{Ri - Rf}{\beta i}$$

The Sharpe's Single Index specifically measures the "excess return to beta" ratio for each stock under consideration. This is shown in the following table (Table 3).

Company Name	Stock Return R_i	Risk free rate R_f	Beta (β_i)	R_i-R_f	Sharpe's Single Index [(R_i-R_f)/β_i]
ACC	0.135	0.081	0.814	0.054	0.066
Ambuja Cem.	0.191	0.081	0.951	0.110	0.116
Asian Paints	0.381	0.081	0.567	0.300	0.529
Axis Bank	0.236	0.081	1.494	0.155	0.103
B H E L	-0.217	0.081	1.242	-0.298	-0.240
B P C L	0.318	0.081	0.821	0.236	0.288
Bajaj Auto	0.182	0.081	0.690	0.101	0.146
Bank of Baroda	0.058	0.081	1.173	-0.023	-0.019
Bharti Airtel	0.072	0.081	0.838	-0.010	-0.012
Cairn India	-0.115	0.081	0.758	-0.196	-0.259
Cipla	0.181	0.081	0.518	0.100	0.192
Coal India	0.019	0.081	0.669	-0.063	-0.093
Dr Reddy's Labs	0.283	0.081	0.454	0.202	0.445
GAIL (India)	-0.051	0.081	0.673	-0.133	-0.197
Grasim Inds	0.080	0.081	0.744	-0.001	-0.001
Housing Dev.	0.215	0.081	1.118	0.133	0.119
HCL Technologies	0.439	0.081	0.743	0.357	0.481
HDFC Bank	0.265	0.081	1.041	0.184	0.177
Hero Motocorp	0.057	0.081	0.624	-0.024	-0.039
Hind. Unilever	0.371	0.081	0.464	0.290	0.625
Hindalco Inds.	-0.096	0.081	1.488	-0.177	-0.119
I D F C	-0.005	0.081	1.558	-0.086	-0.055
ICICI Bank	0.162	0.081	1.530	0.081	0.053
Idea Cellular	0.308	0.081	0.647	0.227	0.351
IndusInd Bank	0.424	0.081	1.275	0.343	0.269
Infosys	0.103	0.081	0.723	0.021	0.029
ITC	0.260	0.081	0.663	0.178	0.269
Kotak Mah. Bank	0.376	0.081	1.139	0.295	0.259
Larsen & Toubro	0.122	0.081	1.325	0.040	0.031
Lupin	0.483	0.081	0.421	0.402	0.954
M & M	0.228	0.081	0.951	0.147	0.154
Maruti Suzuki	0.313	0.081	0.801	0.232	0.290
NMDC	-0.249	0.081	0.896	-0.330	-0.368
NTPC	-0.095	0.081	0.764	-0.176	-0.230
O N G C	0.042	0.081	0.866	-0.040	-0.046
Power Grid Corpn	0.075	0.081	0.625	-0.006	-0.010
Punjab Natl.Bank	-0.077	0.081	1.213	-0.159	-0.131
Reliance Inds.	-0.053	0.081	1.112	-0.134	-0.121
St Bk of India	0.047	0.081	1.280	-0.035	-0.027
Sun Pharma.Inds.	0.524	0.081	0.542	0.443	0.817
Tata Motors	0.317	0.081	1.382	0.235	0.170
Tata Power Co.	-0.160	0.081	0.980	-0.242	-0.247

Tata Steel	-0.159	0.081	1.356	-0.240	-0.177
TCS	0.343	0.081	0.716	0.261	0.365
Tech Mahindra	0.342	0.081	0.667	0.261	0.390
UltraTech Cem.	0.296	0.081	0.722	0.215	0.297
Vedanta	-0.210	0.081	1.319	-0.291	-0.221
Wipro	0.085	0.081	0.627	0.004	0.006
Yes Bank	0.316	0.081	1.621	0.235	0.145
Zee Entertainment	0.211	0.081	0.666	0.129	0.194

Construction of the Optimal Portfolio

Having ranked all the stocks from highest to lowest, we have calculated C_i for all the stocks according to the ranked order, using the following formula:

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{i} \frac{(R_i - R_F) * \beta_i}{\sigma_{e_i}^2}}{1 + \sigma_m^2 \sum_{i=1}^{i} \frac{\beta_i^2}{\sigma_{e_i}^2}}$$

Next we find out the optimum C_i that is C^* (Cut-off point). Upto C^*, all the stocks under review have excess return to beta above C^*, i.e.

$$\frac{Ri - Rf}{\beta i} \geq C^*$$

And, after that point all the securities have excess return to beta below C^* i.e.

$$\frac{Ri - Rf}{\beta i} \leq C^*$$

This is depicted in the following table (Table 4)

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Company Name	Sharpe's Single Index	Ri-Rf	Beta (βi)	Unsystematic risk (σei^2)	Variance of market return (annually) (σm^2)	(Ri-Rf)*βi/σei^2	∑(Ri-Rf)*βi/σei^2	(σm^2)∑(Ri-Rf)*βi/σei^2	(βi^2)/σei^2	∑(βi^2)/σei^2	1+(σm^2)∑(βi^2)/σei^2	Ci= (σm^2)∑((Ri-Rf)*βi/σei^2)/(1+(σm^2)∑(βi^2)/σei^2)
Lupin	0.954	0.402	0.421	8.646	4.133	0.020	0.020	0.081	0.021	0.021	1.085	0.0745
Sun Pharma	0.817	0.443	0.542	8.884	4.133	0.027	0.047	0.192	0.033	0.054	1.221	0.1576
HUL	0.625	0.290	0.464	8.190	4.133	0.016	0.063	0.260	0.026	0.080	1.330	0.1957
Asian Paints	0.529	0.300	0.567	8.206	4.133	0.021	0.084	0.346	0.039	0.119	1.492	0.2318
HCL Tech.	0.481	0.357	0.743	10.608	4.133	0.025	0.109	0.449	0.052	0.171	1.707	0.2632
Dr Reddy's Labs	0.445	0.202	0.454	6.805	4.133	0.013	0.122	0.505	0.030	0.201	1.832	0.2756
Tech Mahindra	0.390	0.261	0.667	11.690	4.133	0.015	0.137	0.566	0.038	0.239	1.989	0.2847
TCS	0.365	0.261	0.716	8.363	4.133	0.022	0.159	0.659	0.061	0.301	2.243	0.2938
Idea Cellular	0.351	0.227	0.647	16.442	4.133	0.009	0.168	0.696	0.025	0.326	2.348	0.2963
UltraTech Cem.	0.297	0.215	0.722	8.196	4.133	0.019	0.187	0.774	0.064	0.390	2.611	0.2965
Maruti Suzuki	0.290	0.232	0.801	9.237	4.133	0.020	0.207	0.857	0.069	0.459	2.898	0.2958
B P C L	0.288	0.236	0.821	13.631	4.133	0.014	0.222	0.916	0.049	0.509	3.102	0.2953
ITC	0.269	0.178	1.275	6.536	4.133	0.035	0.256	1.060	0.249	0.757	4.130	0.2566
IndusInd Bank	0.269	0.343	0.663	10.866	4.133	0.021	0.277	1.146	0.040	0.798	4.298	0.2667
Kotak Mah. Bank	0.259	0.295	1.139	7.295	4.133	0.046	0.323	1.337	0.178	0.976	5.032	0.2656
Zee Ent.	0.194	0.129	0.666	12.616	4.133	0.007	0.330	1.365	0.035	1.011	5.178	0.2636
Cipla	0.192	0.100	0.518	7.396	4.133	0.007	0.337	1.394	0.036	1.047	5.328	0.2616
HDFC Bank	0.177	0.184	1.041	3.970	4.133	0.048	0.385	1.593	0.273	1.320	6.456	0.2468
Tata Motors	0.170	0.235	1.382	11.928	4.133	0.027	0.413	1.706	0.160	1.480	7.118	0.2396
M & M	0.154	0.147	0.951	8.658	4.133	0.016	0.429	1.772	0.104	1.585	7.549	0.2347
Bajaj Auto	0.146	0.101	0.690	7.134	4.133	0.010	0.439	1.812	0.067	1.651	7.825	0.2316

Company Name	Sharpe's Single Index	Ri-Rf	Beta (βi)	Unsystematic risk (σei^2)	Variance of market return (annually) (σm^2)	(Ri-Rf)*βi /σei^2	∑(Ri-Rf)*βi/σei^2	(σm^2)∑(Ri-Rf)*βi /σei^2	(βi^2)/σei^2	∑(βi^2)/σei^2	1+(σm^2)∑(βi^2)/ σei^2	Ci= (σm^2)∑((Ri-Rf)*βi/σei^2)/ (1+(σm^2) ∑(βi^2)/σei^2)
Yes Bank	0.145	0.235	1.621	14.397	4.133	0.026	0.465	1.922	0.182	1.834	8.579	0.2240
Housing Dev.	0.119	0.133	1.118	5.955	4.133	0.025	0.490	2.025	0.210	2.044	9.447	0.2144
Ambuja Cem.	0.116	0.110	0.951	10.754	4.133	0.010	0.500	2.065	0.084	2.128	9.794	0.2109
Axis Bank	0.103	0.155	1.494	8.726	4.133	0.026	0.526	2.175	0.256	2.384	10.852	0.2004
ACC	0.066	0.054	0.814	6.997	4.133	0.006	0.532	2.200	0.095	2.479	11.244	0.1957
ICICI Bank	0.053	0.081	1.530	5.587	4.133	0.022	0.555	2.292	0.419	2.898	12.975	0.1766
Larsen & Toubro	0.031	0.040	1.325	6.916	4.133	0.008	0.562	2.324	0.254	3.151	14.024	0.1657
Infosys	0.029	0.021	0.723	9.858	4.133	0.002	0.564	2.330	0.053	3.204	14.243	0.1636
Wipro	0.006	0.004	0.627	9.082	4.133	0.000	0.564	2.331	0.043	3.247	14.421	0.1617
Grasim Inds	-0.001	-0.001	0.744	7.918	4.133	0.000	0.564	2.331	0.070	3.317	14.710	0.1585
Power Grid Corpn	-0.010	-0.006	0.625	5.694	4.133	-0.001	0.563	2.328	0.069	3.386	14.993	0.1553
Bharti Airtel	-0.012	-0.010	0.838	11.963	4.133	-0.001	0.563	2.325	0.059	3.445	15.236	0.1526
Bank of Baroda	-0.019	-0.023	1.173	11.595	4.133	-0.002	0.560	2.316	0.119	3.563	15.726	0.1473
St Bk of India	-0.027	-0.035	1.280	7.860	4.133	-0.006	0.555	2.292	0.209	3.772	16.588	0.1382
Hero Motocorp	-0.039	-0.024	0.624	10.453	4.133	-0.001	0.553	2.287	0.037	3.809	16.742	0.1366
O N G C	-0.046	-0.040	0.866	9.765	4.133	-0.004	0.550	2.272	0.077	3.886	17.060	0.1332
I D F C	-0.055	-0.086	1.558	11.542	4.133	-0.012	0.538	2.224	0.210	4.096	17.929	0.1240
Coal India	-0.093	-0.063	0.669	11.075	4.150	-0.004	0.534	2.218	0.040	4.137	18.168	0.1221
Hindalco Inds.	-0.119	-0.177	1.488	14.136	4.133	-0.019	0.516	2.131	0.157	4.293	18.743	0.1137
Reliance Inds.	-0.121	-0.134	1.112	5.116	4.133	-0.029	0.486	2.011	0.241	4.535	19.741	0.1018
Punjab Natl.Bank	-0.131	-0.159	1.213	10.623	4.133	-0.018	0.468	1.936	0.138	4.673	20.314	0.0953

Company Name	Sharpe's Single Index	Ri-Rf	Beta (βi)	Unsystematic risk (σei^2)	Variance of market return (annually) (σm^2)	(Ri-Rf)*βi/σei^2	∑(Ri-Rf)*βi/σei^2	(σm^2)∑(Ri-Rf)*βi/σei^2	(βi^2)/σei^2	∑(βi^2)/σei^2	1+(σm^2)∑(βi^2)/σei^2	Ci= (σm^2)∑((Ri-Rf)*βi/σei^2)/(1+(σm^2)∑(βi^2)/σei^2)
Tata Steel	-0.177	-0.240	1.356	9.809	4.133	-0.033	0.435	1.799	0.187	4.861	21.088	0.0853
GAIL (India)	-0.197	-0.133	0.673	8.328	4.133	-0.011	0.425	1.754	0.054	4.915	21.313	0.0823
Vedanta	-0.221	-0.291	1.319	17.099	4.133	-0.022	0.402	1.662	0.102	5.017	21.733	0.0765
NTPC	-0.230	-0.176	0.764	7.745	4.133	-0.017	0.385	1.590	0.075	5.092	22.045	0.0721
B H E L	-0.240	-0.298	1.242	15.085	4.133	-0.025	0.360	1.489	0.102	5.194	22.467	0.0663
Tata Power Co.	-0.247	-0.242	0.980	10.715	4.133	-0.022	0.338	1.397	0.090	5.284	22.837	0.0612
Cairn India	-0.259	-0.196	0.758	9.394	4.133	-0.016	0.322	1.332	0.061	5.345	23.090	0.0577
NMDC	-0.368	-0.330	0.896	10.883	4.133	-0.027	0.295	1.220	0.074	5.419	23.394	0.0521

From the above table, it can be seen that from top to bottom, the C_i values first increase, reach a maximum value and then decrease. This maximum value of C_i is the cut-off point C*. In this case, C* = 0.2965. From the table above, it is observed that the stocks ranked above C* have higher excess returns to beta than their respective C_i values.

Thus the **top ten securities** which make it to the optimal equity portfolio are as follows:

- Asian Paints
- Lupin Limited
- Sun Pharmaceuticals Industries Limited
- Dr. Reddy's Laboratories Limited
- HCL Technologies
- Hindustan Unilever
- Tech Mahindra
- TCS
- Idea Cellular
- Ultra Tech Cement

Our next objective is to find out the exact proportion to be invested in each of the ten stocks mentioned above. The proportion invested in the ith security is as follows:

$$X_i = \frac{Z_i}{\sum_{i=1}^{N} Z_i}$$

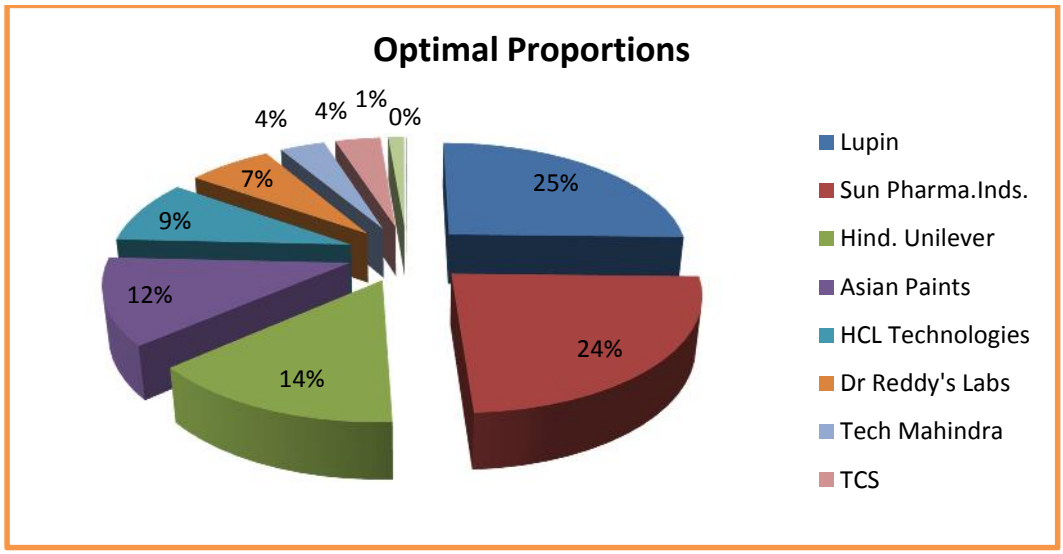
where

$$Z_i = \frac{\beta_i}{\sigma_{e_i}^2}\left(\frac{R_i - R_F}{\beta_i} - C^*\right)$$

The calculation is shown in the following table (Table 5):

Company Name	**Beta (β_i)**	**Unsystematic risk (σ_{ei}^2)**	**Sharpe's Single Index [(R_i-R_f)/β_i]**	**C_i**	**β_i/σ_{ei}^2**	**((R_i-R_f)/β_i)-C_i**	**Z_i=($\beta_i/\sigma ei$^2)*((R_i-R_f)/βi)-C_i)**	**Proportion [X_i=$Z_i/\sum Z_i$]**
Lupin	0.421	8.646	0.954	0.0745	0.049	0.879	0.043	**0.254**
Sun Pharma	0.542	8.884	0.817	0.1576	0.061	0.660	0.040	**0.238**
Hind. Unilever	0.464	8.190	0.625	0.1957	0.057	0.429	0.024	**0.144**
Asian Paints	0.567	8.206	0.529	0.2318	0.069	0.297	0.021	**0.121**
HCL Tech.	0.743	10.608	0.481	0.2632	0.070	0.218	0.015	**0.090**
Dr Reddy's Labs	0.454	6.805	0.445	0.2756	0.067	0.169	0.011	**0.067**
Tech Mahindra	0.667	11.690	0.390	0.2847	0.057	0.106	0.006	**0.036**
TCS	0.716	8.363	0.365	0.2938	0.086	0.071	0.006	**0.036**
Idea Cellular	0.647	16.442	0.351	0.2963	0.039	0.054	0.002	**0.013**
UltraTech Cem.	0.722	8.196	0.297	0.2965	0.088	0.001	0.000	**0.001**
						$\sum Z_i$	**0.169**	**1.000**

The portfolio mix and its proportions can be illustrated in the diagram below (Figure 1)



Risk-Return Analysis

The Portfolio Return is given by

$$R_p = \sum_i X_i R_i$$

And, the Portfolio Beta is given by

$$\beta_p = \sum_i X_i \beta_i$$

where, Xi, Ri and βi represents the proportion, Return and Beta for the ith stock.
The calculations are shown in the following table (Table 6):

Company Name	Stock Return (R_i)	Beta (β_i)	Variance of Stock return (annually) (Total Risk) (%)	Proportion (X_i)	X_i*R_i	Portfolio Return ($\sum X_i*R_i$)	$X_i*\beta_i$	Portfolio Beta ($\sum X_i*\beta_i$)
Lupin	0.483	0.421	9.379	0.254	0.123		0.107	
Sun Pharma.Inds.	0.524	0.542	10.097	0.238	0.125		0.129	
						0.434		**0.528**
Hind. Unilever	0.371	0.464	9.079	0.144	0.053		0.067	
Asian Paints	0.381	0.567	9.534	0.121	0.046		0.069	
HCL Tech.	0.439	0.743	12.892	0.090	0.040		0.067	
Dr Reddy's Labs	0.283	0.454	7.656	0.067	0.019		0.030	
Tech Mahindra	0.342	0.667	13.532	0.036	0.012		0.024	
TCS	0.343	0.716	10.480	0.036	0.012		0.026	
Idea Cellular	0.308	0.647	18.172	0.013	0.004		0.008	
UltraTech Cem.	0.296	0.722	10.353	0.001	0.0002		0.0004	

Portfolio Variance is given by

$$\sigma_p^2 = \sum_i X_i^2 \sigma_i^2 + 2\sum_i \sum_j X_i X_j Cov_{i,j}$$

To calculate the portfolio variance, it is necessary to find all the covariance combinations between individual stocks as shown in the following table (Table 7):

Portfolio selection from the CNX nifty stocks based on sharpe's single index model

Company Name	Lupin	Sun Pharma	HUL	Asian Paints	HCL Tech	Dr Reddy's	Tech Mahindra	TCS	Idea Cellular	Ultra Tech
Lupin	**9.3792**									
Sun Pharma	2.6856	**10.0967**								
HUL	1.0072	1.5925	**9.0795**							
Asian Paints	1.4086	1.6889	1.4933	**9.5338**						
HCL Tech	1.6571	2.1260	1.6015	1.0811	**12.8916**					
Dr Reddy's	2.1655	2.5026	1.7636	0.9294	1.9824	**7.6555**				
Tech Mahindra	1.7738	1.7847	1.1793	1.5442	4.7745	2.0819	**13.5315**			
TCS	1.3148	1.9524	1.4226	1.3369	6.0859	1.7795	4.0573	**10.4796**		
Idea Cellular	1.7961	1.6562	1.2877	1.8884	1.2616	0.9696	2.5808	1.5306	**18.1719**	
UltraTech	1.7018	1.5889	1.1823	2.6014	1.3974	0.9048	1.7638	1.5177	2.1325	**10.3528**

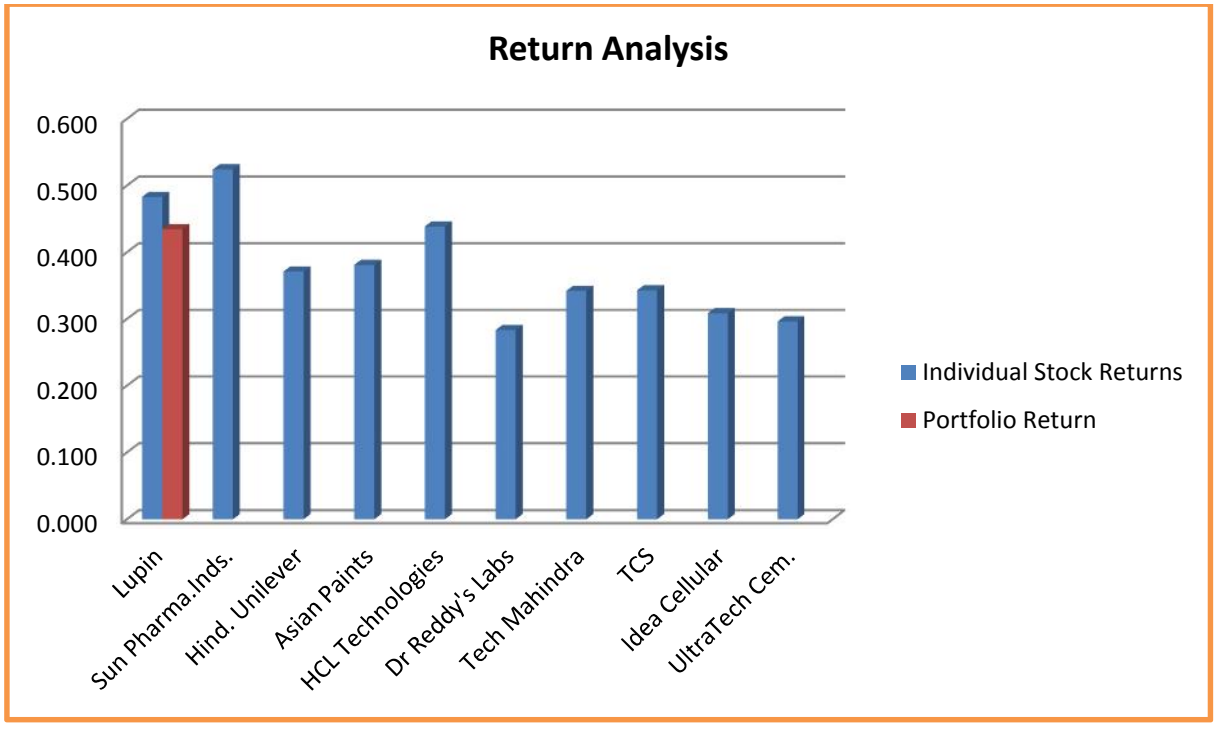
The numerical figures which are not in bold are the covariance of the stocks between each other. The bold numerical figures are the variances.
Now,

$$\sum_i X_i^2 \sigma_i^2 = 1.6799$$

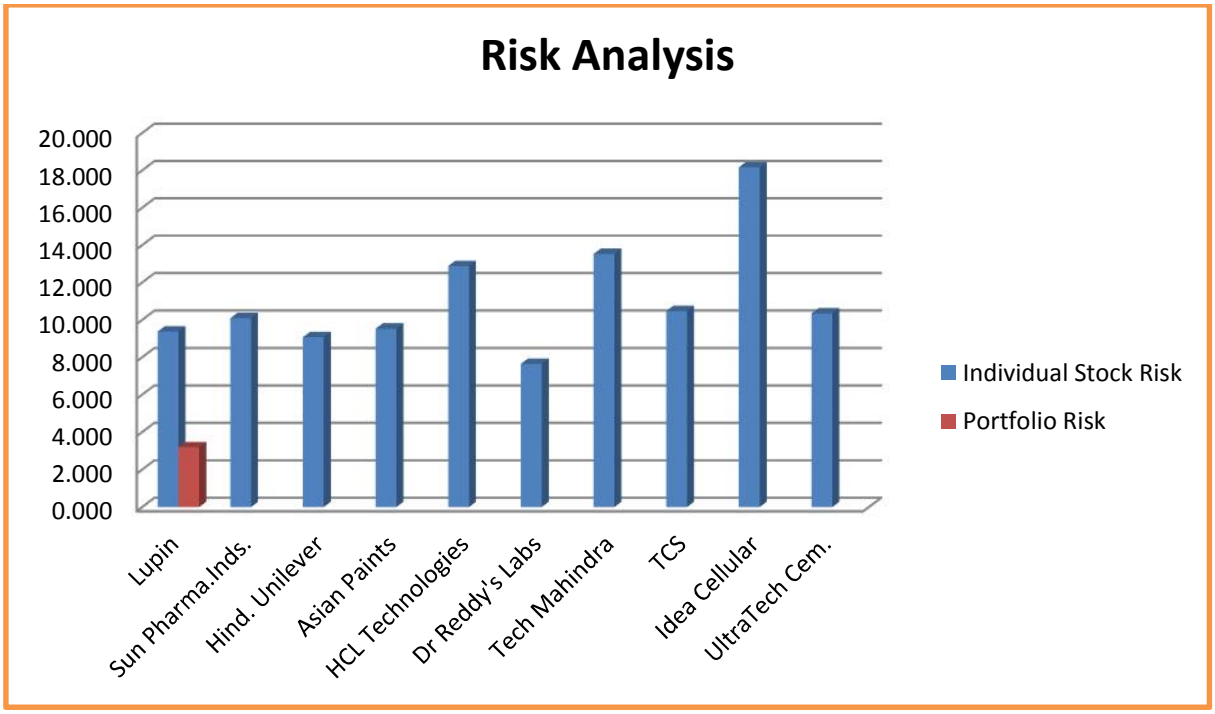
$$\sum_i \sum_j X_i X_j Cov_{i,j} = 0.7593$$

Therefore, Portfolio Variance = 3.198
This is the minimum among that of all the individual stocks.

Portfolio diversification refers to the strategy of **reducing risk** by combining different types of assets in a portfolio. The portfolio return remains high whereas the portfolio risk (variance) becomes very low in case of an optimal (efficient) portfolio. This efficient portfolio is also known as the **Minimum Variance Portfolio**, as for every other proportion combination of these 10 stocks, the portfolio variance will be higher than the current portfolio variance which is the minimum risk.
The underlying assumption is that the individual is risk averse that is for a given return, he will always try to reduce the risk. The diagram below (Figure 2) illustrates how the portfolio return remains high as compared to that of the individual stocks.



The diagram below (Figure 3) illustrates how the portfolio risk is minimized in case of the optimal portfolio as compared to that of the individual stocks.



RECOMMENDATIONS TO THE INDIVIDUAL/ INSTITUTIONAL INVESTOR

The companies chosen for study have shown much volatility during the period of 2010-2015 as shown by their variance analysis. Even though the companies were chosen from all the sectors constituting NIFTY index, the optimal equity portfolio consists of companies of securities belonging to only to the Manufacturing, Pharmaceutical and the IT sector. The risk component of individual securities comprises both the Systematic as well as the Unsystematic risk. Hence it is prudent for a risk-averse investor to diversify his investments over an optimal portfolio which reduces the unsystematic component of the risk. This result has been well validated as shown by the minimum variance of the above constructed portfolio.

Equity constitutes a major segment in the total investment portfolio of most Individual/Institutional investor and has always been the mainstay of the investment operations in India. Many of the NIFTY stocks, which are the top fifty in terms of market capitalization, form an essential component of the equity portfolio of most investors. Hence this study recommends the equity investor to focus his investments in the Manufacturing, Pharmaceutical and the IT sector as indicated by the results of the optimal equity portfolio.

LIMITATIONS OF THE STUDY

- The study has been restricted to the equity market segment only.
- Historic data (for the last 5 years) has been used for the analysis which may not truly represent the future.
- The recent volatility experienced in the stock markets which has been reflected in the stock prices considered in the study might not produce very accurate results.
- This study considers investments only in risky assets. In practice, an investment portfolio considers a combination of both risky as well as risk free assets. Hence solely an "equity portfolio" may not be all encompassing.
- Along with Sharpe's Single Index Model it is wise for investment analysts to perform technical analysis and fundamental analysis of the company, industry as well as the economy as a whole before investing in that particular company.

CONCLUSION

As discussed previously, equity constitutes a major segment in the total investment portfolio of most Indian Investors and has always been the mainstay of their investment activities. This report specifically portrays the findings and recommendations to an individual/institutional investor, based on the research carried out within the boundaries of the Nifty stocks selected.

It is to be noted is that this research finding should not be considered by an investor as a standalone all-encompassing yardstick while making investment decisions. In order to get more accurate and detailed results, these research findings must be clubbed with the findings of the relevant fundamental and technical analysis carried out by the respective fundamental and technical analysts or by the investor himself.

Another important aspect which should be highlighted is that the equity market is never static. It is always a dynamic ever changing environment for equity investors. At this point of time, these research findings which are based on the latest prices of the NIFTY stocks and that of the index itself are bound to give accurate results. However, in future, as time will pass these research findings will soon be outdated since then, they will not be based on the latest data. Hence it is important for the investor to run this research using the latest available stock data as and when required for carrying out analysis for future time periods.

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